

# Natural Language Processing

— Context-Free Grammars and Chart Parsing —

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# Reminding Ourselves — Context-Free Grammars

- Formally, a *context-free grammar* (CFG) is a quadruple:  $\langle C, \Sigma, P, S \rangle$
- $C$  is the set of categories (aka *non-terminals*), e.g.  $\{S, NP, VP, V\}$ ;
- $\Sigma$  is the vocabulary (aka *terminals*), e.g.  $\{\text{Juan, nieve, amó, en}\}$ ;
- $P$  is a set of category rewrite rules (aka *productions*), e.g.

S  $\rightarrow$  NP VP  
VP  $\rightarrow$  V NP  
NP  $\rightarrow$  Juan  
NP  $\rightarrow$  nieve  
V  $\rightarrow$  amó

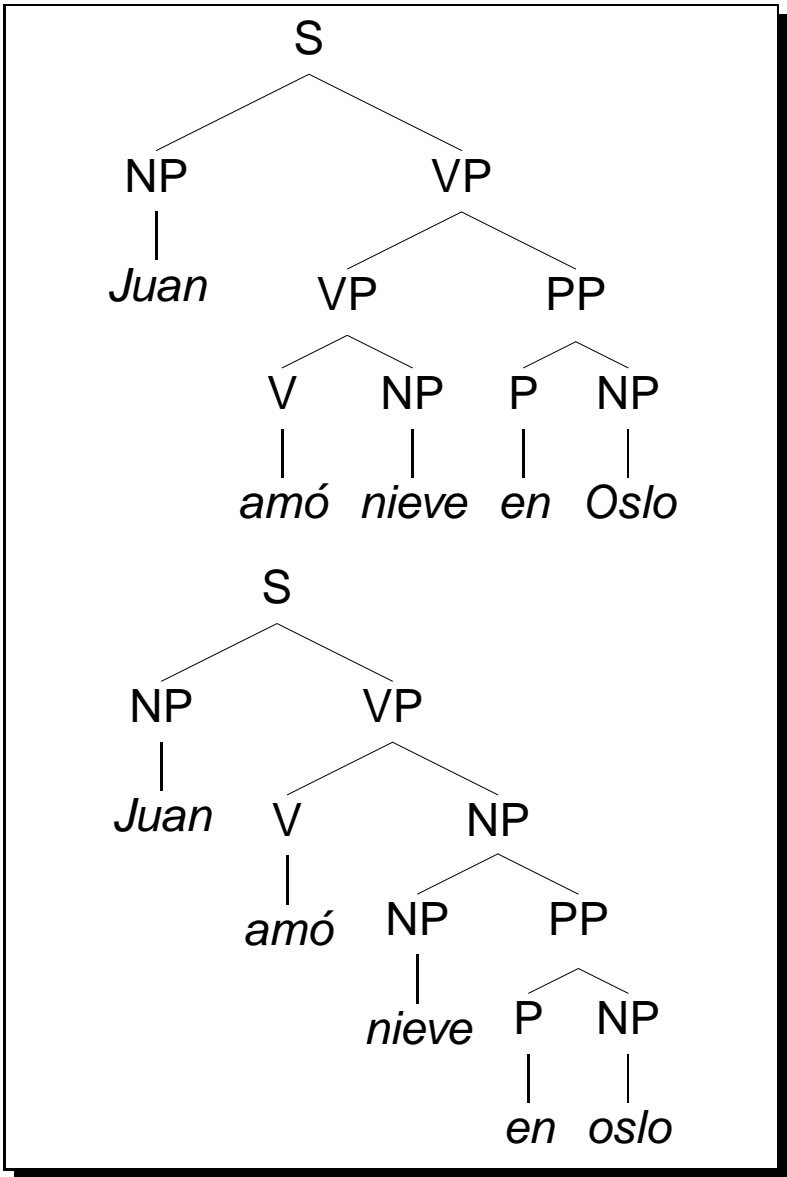
- $S \in C$  is the *start symbol*, a filter on complete ('sentential') results;
- for each rule ' $\alpha \rightarrow \beta_1, \beta_2, \dots, \beta_n$ '  $\in P$ :  $\alpha \in C$  and  $\beta_i \in C \cup \Sigma$ ;  $1 \leq i \leq n$ .



# Parsing: Recognizing the Language of a Grammar

- $S \rightarrow NP VP$
- $VP \rightarrow V NP$
- $VP \rightarrow VP PP$
- $NP \rightarrow NP PP$
- $PP \rightarrow P NP$
- $NP \rightarrow \text{Juan} \mid \text{nieve} \mid \text{Oslo}$
- $V \rightarrow \text{amó}$
- $P \rightarrow \text{en}$

- All Complete Derivations**
- are rooted in the start symbol  $S$ ;
  - label internal nodes with categories  $\in C$ , leafs with words  $\in \Sigma$ ;
  - instantiate a grammar rule  $\in P$  at each local subtree of depth one.



# Top-Down vs. Bottom-Up Parsing

## Top-Down (Goal-Oriented)

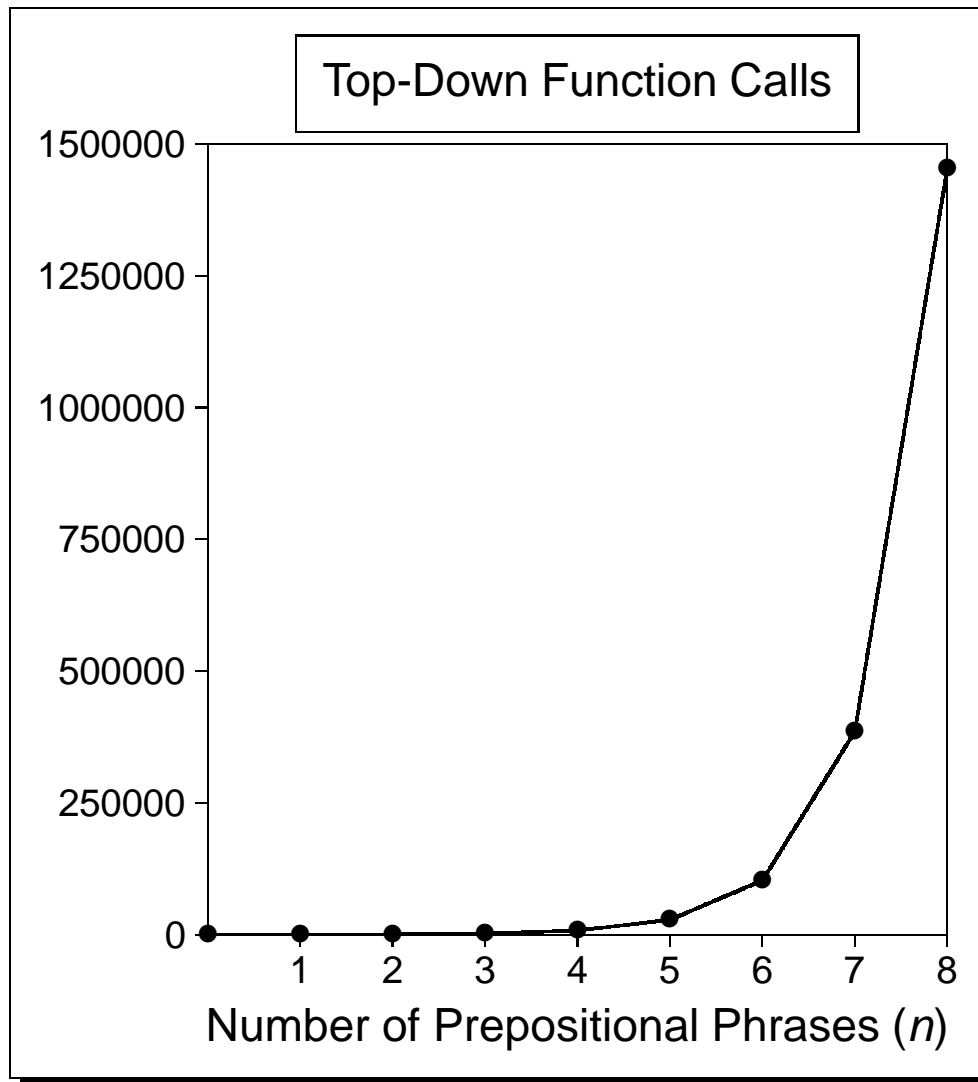
- Left recursion (e.g. the 'VP  $\rightarrow$  VP PP' rule) causes infinite recursion;
  - grammar conversion techniques (eliminating left recursion) exist, but will often be undesirable for natural language processing applications;
- assume bottom-up as basic search strategy for NLP applications.

## Bottom-Up (Data-Oriented)

- unary (left-recursive) rules (e.g. 'NP  $\rightarrow$  NP') would still be problematic;
- lack of parsing goal: compute all possible derivations for, say, the input *adores snow*; however, it is ultimately rejected since it is not sentential;
- availability of partial analyses desirable for, at least, some applications.



# Quantifying the Complexity of the Parsing Task



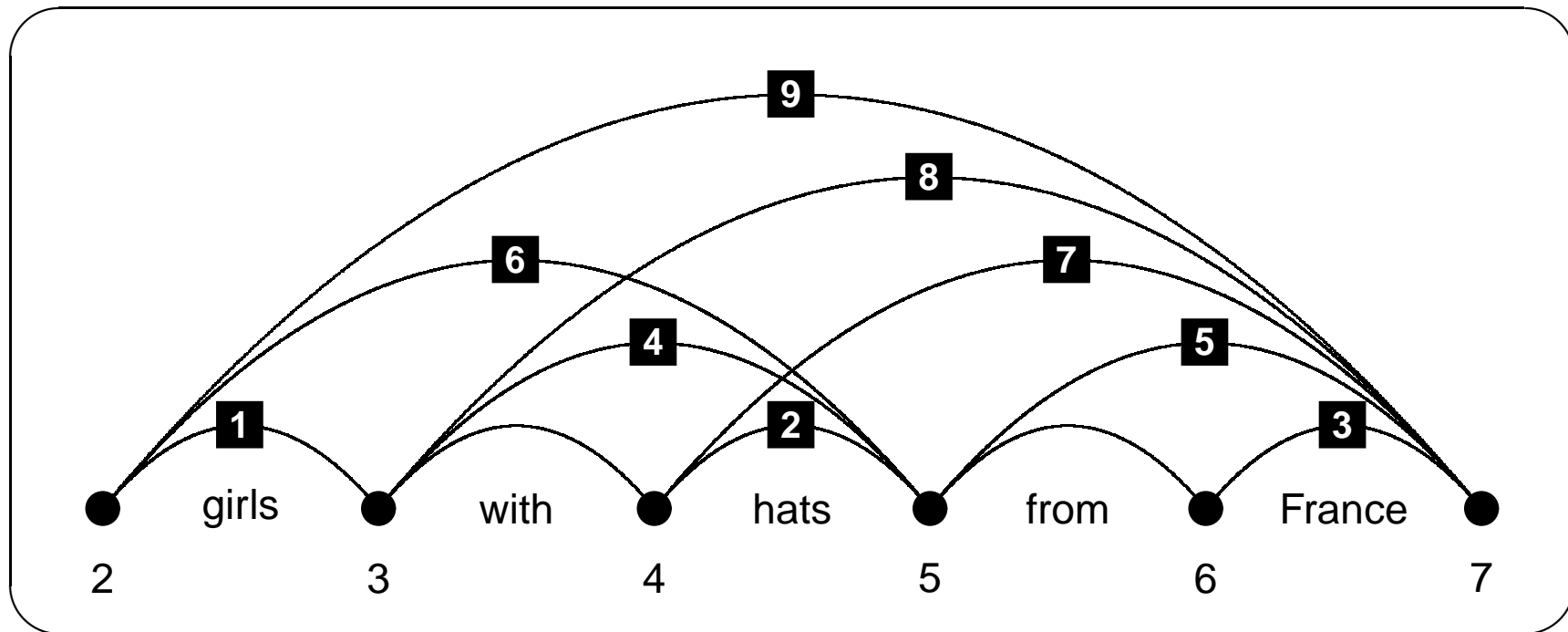
*Kim adores snow (in Oslo)<sup>n</sup>*

<i>n</i>	trees	calls
0	1	46
1	2	170
2	5	593
3	14	2,093
4	42	7,539
5	132	27,627
6	429	102,570
7	1430	384,566
8	4862	1,452,776
⋮	⋮	⋮



# Using the Chart to Bound Ambiguity

- For many substrings, multiple ways of deriving the same category;
- NPs: **1** | **2** | **3** | **6** | **7** | **9**; PPs: **4** | **5** | **8**; **9**  $\equiv$  **1** + **8** | **6** + **5**;
- *parse forest* — a single item represents multiple trees [Billot & Lang, 89].



# Dynamic Programming: Chart Parsing

## Basic Notions

- Use *chart* to record partial analyses, indexing them by string positions;
- count inter-word vertices; CKY: chart row is *start*, column *end* vertex;
- treat multiple ways of deriving the same category for same substring as *equivalent*; pursue only once when combining with other constituents.

## Key Benefits

- Dynamic programming (memoization): avoid recomputation of results;
- efficient indexing of constituents: no search by start or end positions;
- compute *parse forest* with exponential 'extension' in *polynomial* time.



# The CKY (Cocke, Kasami, & Younger) Algorithm

```

for ( $0 \leq i < |input|$ ) do
   $chart_{[i,i+1]} \leftarrow \{\alpha \mid \alpha \rightarrow input_i \in P\};$ 
for ( $1 \leq l < |input|$ ) do
  for ( $0 \leq i < |input| - l$ ) do
    for ( $1 \leq j \leq l$ ) do
      if ( $\alpha \rightarrow \beta_1 \beta_2 \in P \wedge \beta_1 \in chart_{[i,i+j]} \wedge \beta_2 \in chart_{[i+j,i+l+1]}$ ) then
         $chart_{[i,i+l+1]} \leftarrow chart_{[i,i+l+1]} \cup \{\alpha\};$ 

```

$[0,2] \leftarrow [0,1] + [1,2]$

...

$[0,5] \leftarrow [0,1] + [1,5]$

$[0,5] \leftarrow [0,2] + [2,5]$

$[0,5] \leftarrow [0,3] + [3,5]$

$[0,5] \leftarrow [0,4] + [4,5]$

	1	2	3	4	5
0	NP		S		S
1		V	VP		VP
2			NP		NP
3				P	PP
4					NP





# Limitations of the CKY Algorithm

## Built-In Assumptions

- *Chomsky Normal Form* grammars:  $\alpha \rightarrow \beta_1\beta_2$  or  $\alpha \rightarrow \gamma$  ( $\beta_i \in C$ ,  $\gamma \in \Sigma$ );
- breadth-first (aka exhaustive): always compute all values for each cell;
- rigid control structure: bottom-up, left-to-right (one diagonal at a time).

## Generalized Chart Parsing

- Liberate order of computation: no assumptions about earlier results;
- *active edges* encode partial rule instantiations, ‘waiting’ for additional (adjacent and passive) constituents to complete:  $[1, 2, VP \rightarrow V \bullet NP]$ ;
- parser can fill in chart cells in *any* order and guarantee completeness.



# Generalized Chart Parsing

- The *chart* is a two-dimensional matrix of *edges* (aka chart items);
- an edge is a (possibly partial) rule instantiation over a substring of input;
- the chart indexes edges by start and end string position (aka vertices);
- dot in rule RHS indicates degree of completion:  $\alpha \rightarrow \beta_1 \dots \beta_{i-1} \bullet \beta_i \dots \beta_n$
- *active* edges (aka *incomplete* items) — partial RHS:  $[1, 2, VP \rightarrow V \bullet NP]$ ;
- *passive* edges (aka *complete* items) — full RHS:  $[1, 3, VP \rightarrow V NP \bullet]$ ;

## The Fundamental Rule

$$[x, y, \alpha \rightarrow \beta_1 \dots \beta_{i-1} \bullet \beta_i \dots \beta_n] + [y, z, \beta_i \rightarrow \gamma^+ \bullet] \\ \mapsto [x, z, \alpha \rightarrow \beta_1 \dots \beta_i \bullet \beta_{i+1} \dots \beta_n]$$



# An Example of a (Near-)Complete Chart

	1	2	3	4	5
0	$NP \rightarrow NP \bullet PP$ $S \rightarrow NP \bullet VP$ $NP \rightarrow kim \bullet$				$S \rightarrow NP VP \bullet$
1		$VP \rightarrow V \bullet NP$ $V \rightarrow adores \bullet$	$VP \rightarrow VP \bullet PP$ $VP \rightarrow V NP \bullet$		$VP \rightarrow VP \bullet PP$ $VP \rightarrow VP PP \bullet$ $VP \rightarrow V PP \bullet$
2			$NP \rightarrow NP \bullet PP$ $NP \rightarrow snow \bullet$		$NP \rightarrow NP \bullet PP$ $NP \rightarrow NP PP \bullet$
3				$PP \rightarrow P \bullet NP$ $P \rightarrow in \bullet$	$PP \rightarrow P NP \bullet$
4					$NP \rightarrow NP \bullet PP$ $NP \rightarrow oslo \bullet$

0 *Kim* 1 *adores* 2 *snow* 3 *in* 4 *Oslo* 5



## (Even) More Active Edges

	0	1	2	3
0	$S \rightarrow \bullet NP VP$ $NP \rightarrow \bullet NP PP$ $NP \rightarrow \bullet kim$	$S \rightarrow NP \bullet VP$ $NP \rightarrow NP \bullet PP$ $NP \rightarrow kim \bullet$		$S \rightarrow NP VP \bullet$
1		$VP \rightarrow \bullet VP PP$ $VP \rightarrow \bullet V NP$ $V \rightarrow \bullet adores$	$VP \rightarrow V \bullet NP$ $V \rightarrow adores \bullet$	$VP \rightarrow VP \bullet PP$ $VP \rightarrow V NP \bullet$
2			$NP \rightarrow \bullet NP PP$ $NP \rightarrow \bullet snow$	$NP \rightarrow NP \bullet PP$ $NP \rightarrow snow \bullet$
3				

- Include all grammar rules as *epsilon* edges in each  $chart_{[i,i]}$  cell.
- after initialization, apply *fundamental rule* until fixpoint is reached.



# Our ToDo List: Keeping Track of Remaining Work

## The Abstract Goal

- Any chart parsing algorithm needs to check all pairs of adjacent edges.

## A Naïve Strategy

- Keep iterating through the complete chart, combining all possible pairs, until no additional edges can be derived (i.e. the fixpoint is reached);
- frequent attempts to combine pairs multiple times: deriving ‘duplicates’.

## An Agenda-Driven Strategy

- Combine each pair exactly once, viz. when both elements are available;
- maintain *agenda* of new edges, yet to be checked against chart edges;
- new edges go into agenda first, add to chart upon retrieval from agenda.



# Backpointers: Keeping Track of the Derivation History

	0	1	1	3
0	2: S → • NP VP 1: NP → • NP PP 0: NP → • kim	10: S → 8 • VP 9: NP → 8 • PP 8: NP → kim •		17: S → 8 15 •
1		5: VP → • VP PP 4: VP → • V NP 3: V → • adores	12: VP → 11 • NP 11: V → adores •	16: VP → 15 • PP 15: VP → 11 13 •
2			7: NP → • NP PP 6: NP → • snow	14: NP → 13 • PP 13: NP → snow •
3				

- Use edges to record derivation trees: backpointers to daughters;
- a single edge can represent multiple derivations: backpointer sets.



# Ambiguity Packing in the Chart

## General Idea

- Maintain only one edge for each  $\alpha$  from  $i$  to  $j$  (the ‘representative’);
- record alternate sequences of daughters for  $\alpha$  in the representative.

## Implementation

- Group passive edges into *equivalence classes* by identity of  $\alpha$ ,  $i$ , and  $j$ ;
  - search chart for existing equivalent edge ( $h$ , say) for each new edge  $e$ ;
  - when  $h$  (the ‘host’ edge) exists, *pack*  $e$  into  $h$  to record equivalence;
  - $e$  *not* added to the chart, no derivations with or further processing of  $e$ ;
- *unpacking* multiply out all alternative daughters for all result edges.

