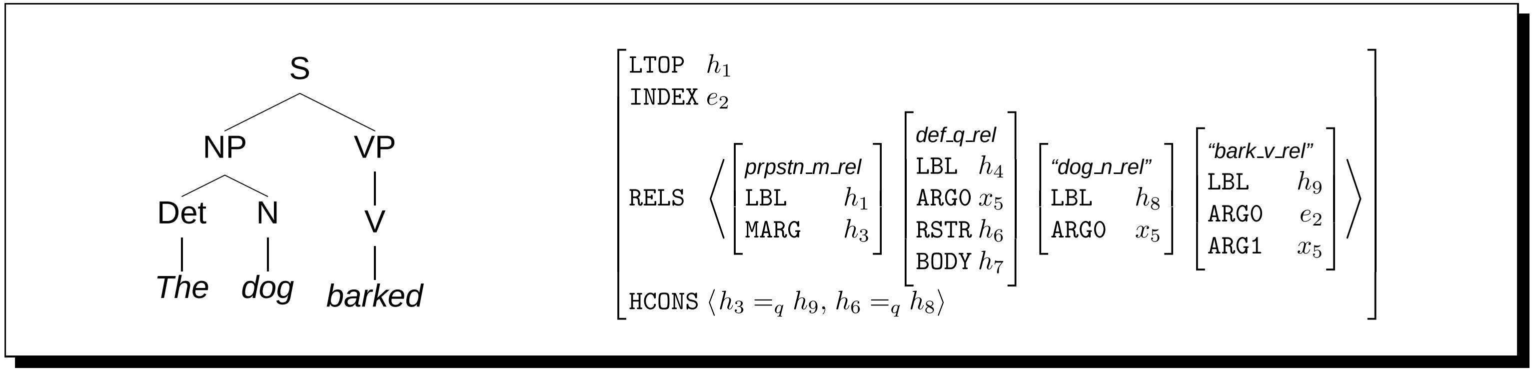

Algorithms for AI and NLP (INF4820 — Parsing)

S $\longrightarrow$ *NP VP*; *NP* $\longrightarrow$ *Det N*; *VP* $\longrightarrow$ *V NP*

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Mildly Mathematically: Context-Free Grammars

- Formally, a *context-free grammar* (CFG) is a quadruple: $\langle C, \Sigma, P, S\rangle$
- C is the set of categories (aka *non-terminals*), e.g. $\{$S, NP, VP, V$\}$;
- Σ is the vocabulary (aka *terminals*), e.g. $\{$Kim, snow, saw, in$\}$;
- P is a set of category rewrite rules (aka *productions*), e.g.

S $\rightarrow$ NP VP
VP $\rightarrow$ V NP
NP $\rightarrow$ Kim
NP $\rightarrow$ snow
V $\rightarrow$ saw

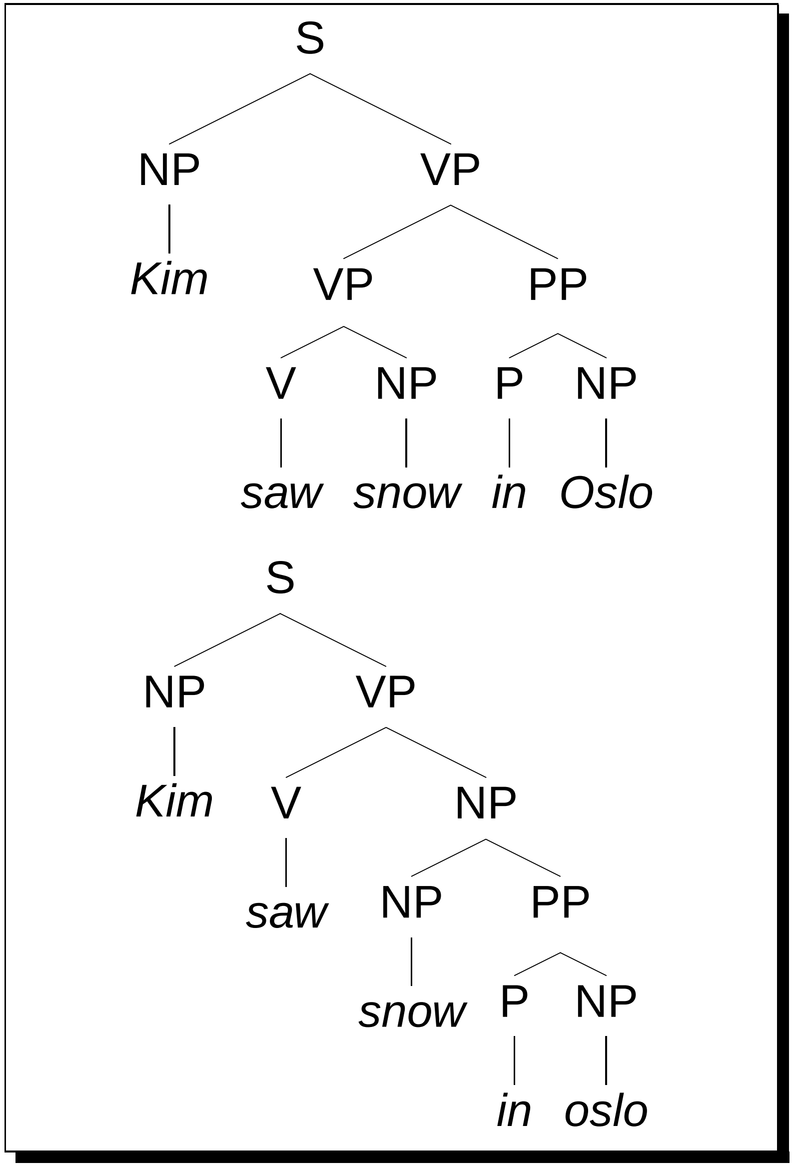
- $S \in C$ is the *start symbol*, a filter on complete ('sentential') results;
- for each rule '$\alpha \rightarrow \beta_1, \beta_2, ..., \beta_n$' $\in P$: $\alpha \in C$ and $\beta_i \in C \cup \Sigma$; $1 \leq i \leq n$.

Parsing: Recognizing the Language of a Grammar

S → NP VP
VP → V | V NP | VP PP
NP → NP PP
PP → P NP
NP → Kim | snow | Oslo
V → saw
P → in

All Complete Derivations

- are rooted in the start symbol S;
- label internal nodes with categories $\in C$, leafs with words $\in \Sigma$;
- instantiate a grammar rule $\in P$ at each local subtree of depth one.



A Simple-Minded Parsing Algorithm

Control Structure

- top-down: given a parsing goal α, use all grammar rules that rewrite α;
- successively instantiate (extend) the right-hand sides of each rule;
- for each β_i in the RHS of each rule, recursively attempt to parse β_i;
- termination: when α is a prefix of the input string, parsing succeeds.

(Intermediate) Results

- Each result records a (partial) tree and remaining input to be parsed;
- complete results consume the full input string and are rooted in S;
- whenever a RHS is fully instantiated, a new tree is built and returned;
- all results at each level are combined and successively accumulated.

A Recursive Descent Parser

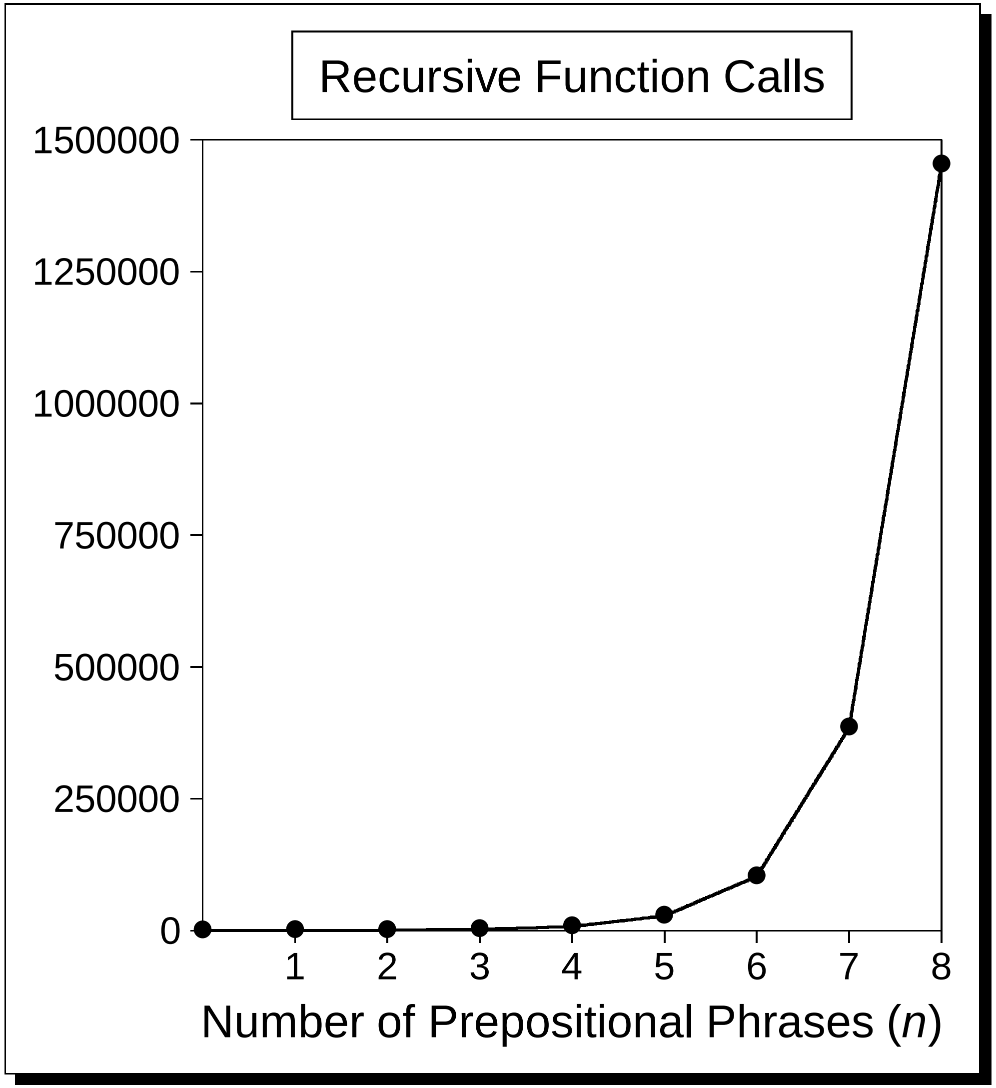
```
(defun parse (input goal)
  (if (equal (first input) goal)
    (list (make-state :tree (first input) :input (rest input)))
    (loop
        for rule in (rules-rewriting goal)
        append (instantiate (rule-lhs rule) nil (rule-rhs rule) input))))
```

```
(defun instantiate (lhs analyzed unanalyzed input)
  (if (null unanalyzed)
    (list (make-state :tree (make-tree :root lhs :daughters analyzed)
                      :input input))
    (loop
        for parse in (parse input (first unanalyzed))
        append (instantiate
                lhs
                (append analyzed (list (state-tree parse)))
                (rest unanalyzed)
                (state-input parse)))))
```

A Closer Look at the Calling Sequence

```
SSP(18): (parse '(kim adored snow) 's)
parse(): input: (KIM ADORED SNOW); goal: S
  parse(): input: (KIM ADORED SNOW); goal: NP
    parse(): input: (KIM ADORED SNOW); goal: KIM
    parse(): input: (KIM ADORED SNOW); goal: SANDY
    parse(): input: (KIM ADORED SNOW); goal: SNOW
  parse(): input: (ADORED SNOW); goal: VP
    parse(): input: (ADORED SNOW); goal: V
      parse(): input: (ADORED SNOW); goal: LAUGHED
      parse(): input: (ADORED SNOW); goal: ADORED
    parse(): input: (ADORED SNOW); goal: V
      parse(): input: (ADORED SNOW); goal: LAUGHED
      parse(): input: (ADORED SNOW); goal: ADORED
    parse(): input: (SNOW); goal: NP
      ...
```

Quantifying the Complexity of the Parsing Task



Kim adores snow (in Oslo)^n

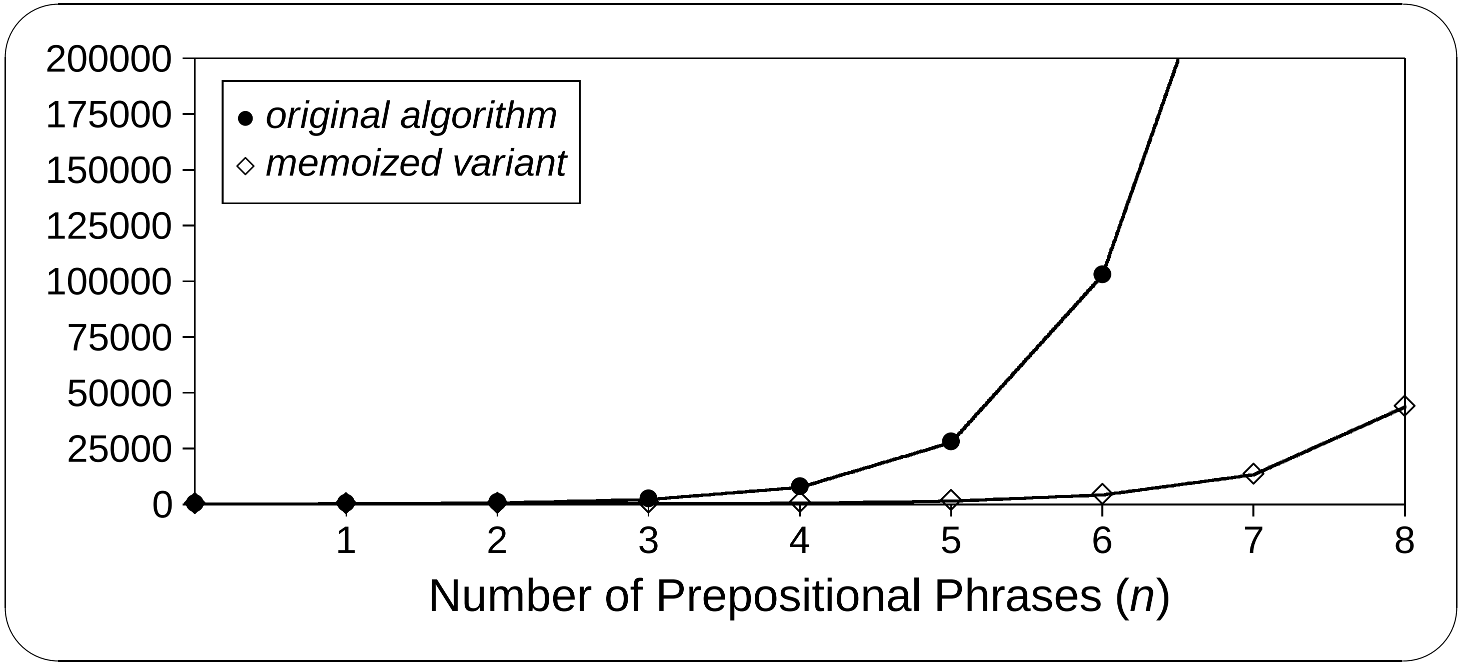
n	**trees**	**calls**
0	1	46
1	2	170
2	5	593
3	14	2,093
4	42	7,539
5	132	27,627
6	429	102,570
7	1430	384,566
8	4862	1,452,776
⋮	⋮	⋮

Memoization: Remember Earlier Results

Dynamic Programming

- The function call `(parse (adored snow) V)` executes two times;
- *memoization* — record `parse()` results for each set of arguments;

→ requires abstract data type, efficient indexing on *input* and *goal*.



Top-Down vs. Bottom-Up Parsing

Top-Down (Goal-Oriented)

- Left recursion (e.g. a rule like 'VP $\rightarrow$ VP PP') causes infinite recursion;
- grammar conversion techniques (eliminating left recursion) exist, but will typically be undesirable for natural language processing applications;

$\rightarrow$ assume bottom-up as basic search strategy for remainder of the course.

Bottom-Up (Data-Oriented)

- unary (left-recursive) rules (e.g. 'NP $\rightarrow$ NP') would still be problematic;
- lack of parsing goal: compute all possible derivations for, say, the input *adores snow*; however, it is ultimately rejected since it is not sentential;
- availability of partial analyses desirable for, at least, some applications.

A Bottom-Up Variant (1 of 2)

- Work upwards from string; successively combine words or phrases into larger phrases;
- use all grammar rules that have the (currently) next input word as β_1 in their RHS;
- recursively attempt to instantiate the remaining part of each rule RHS (β_i; $2 \leq i \leq n$);
- when a rule $\alpha \rightarrow \beta_i^+$ has been completely instantiated, attempt all rules starting in α;
- for each (remaining) input (suffix), derive all trees that span a prefix or all of the input.

```
(defun parse (input)
  (when input
    (loop
        for rule in (rules-starting-in (first input))
        append (instantiate (rule-lhs rule)
                            (list (first (rule-rhs rule)))
                            (rest (rule-rhs rule))
                            (rest input)))))
```

A Bottom-Up Variant (2 of 2)

```
(defun instantiate (lhs analyzed unanalyzed input)
  (if (null unanalyzed)
    (let ((tree (make-tree :root lhs :daughters analyzed)))
      (cons (make-state :tree tree :input input)
            (loop
                for rule in (rules-starting-in lhs)
                append
                  (instantiate (rule-lhs rule)
                               (list tree)
                               (rest (rule-rhs rule))
                               input))))
    (loop
        for state in (parse input)
        when (equal (tree-root (state-tree state))
                    (first unanalyzed))
        append (instantiate lhs
                            (append analyzed (list (state-tree state)))
                            (rest unanalyzed)
                            (state-input state)))))
```

Chart Parsing — Specialized Dynamic Programming

Basic Notions

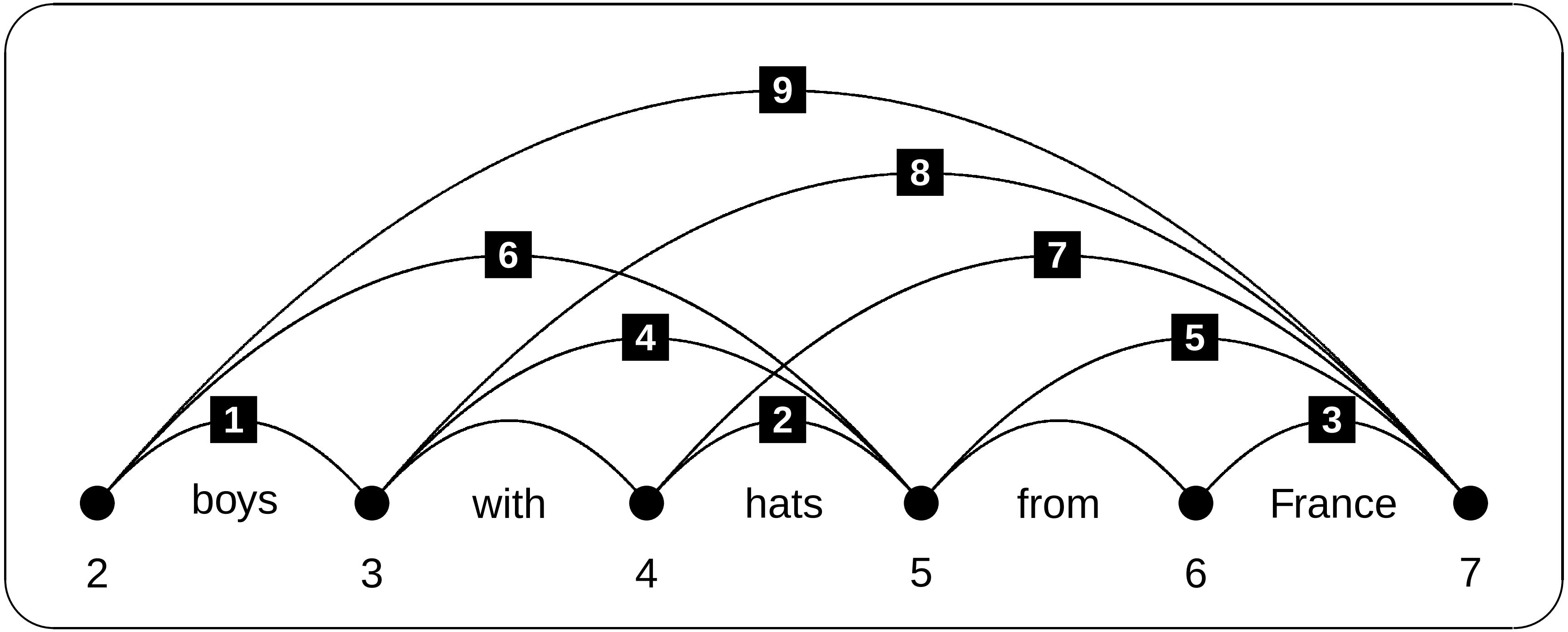
- Use *chart* to record partial analyses, indexing them by string positions;
- count inter-word vertices; CKY: chart row is *start*, column *end* vertex;
- treat multiple ways of deriving the same category for some substring as *equivalent*; pursue only once when combining with other constituents.

Key Benefits

- Dynamic programming (memoization): avoid recomputation of results;
- efficient indexing of constituents: no search by start or end positions;
- compute *parse forest* with exponential 'extension' in *polynomial* time.

Bounding Ambiguity — The Parse Chart

- For many substrings, more than one way of deriving the same category;
- NPs: 1 | 2 | 3 | 6 | 7 | 9; PPs: 4 | 5 | 8; 9 ≡ 1 + 8 | 6 + 5;
- *parse forest* — a single item represents multiple trees [Billot & Lang, 89].



The CKY (Cocke, Kasami, & Younger) Algorithm

for $(0 \leq i < |\textit{input}|)$ do
  $\textit{chart}_{[i,i+1]} \leftarrow \{\alpha \,|\, \alpha \rightarrow \textit{input}_i \in P\}$;
for $(1 \leq l < |\textit{input}|)$ do
  for $(0 \leq i < |\textit{input}| - l)$ do
    for $(1 \leq j \leq l)$ do
      if $(\alpha \rightarrow \beta_1\,\beta_2 \in P \wedge \beta_1 \in \textit{chart}_{[i,i+j]} \wedge \beta_2 \in \textit{chart}_{[i+j,i+l+1]})$ then
        $\textit{chart}_{[i,i+l+1]} \leftarrow \textit{chart}_{[i,i+l+1]} \cup \{\alpha\}$;

$[0,2] \leftarrow [0,1] + [1,2]$
$\ldots$
$[0,5] \leftarrow [0,1] + [1,5]$
$[0,5] \leftarrow [0,2] + [2,5]$
$[0,5] \leftarrow [0,3] + [3,5]$
$[0,5] \leftarrow [0,4] + [4,5]$

	1	2	3	4	5
0	NP		S		S
1		V	VP		VP
2			NP		NP
3				P	PP
4					NP

Limitations of the CKY Algorithm

Built-In Assumptions

- *Chomsky Normal Form* grammars: $\alpha \rightarrow \beta_1\beta_2$ or $\alpha \rightarrow \gamma$ ($\beta_i \in C$, $\gamma \in \Sigma$);
- breadth-first (aka exhaustive): always compute all values for each cell;
- rigid control structure: bottom-up, left-to-right (one diagonal at a time).

Generalized Chart Parsing

- Liberate order of computation: no assumptions about earlier results;
- *active edges* encode partial rule instantiations, 'waiting' for additional (adjacent and passive) constituents to complete: $[1, 2, \text{VP} \rightarrow \text{V} \bullet \text{NP}]$;
- parser can fill in chart cells in *any* order and guarantee completeness.

Generalized Chart Parsing

- The parse *chart* is a two-dimensional matrix of *edges* (aka chart items);
- an edge is a (possibly partial) rule instantiation over a substring of input;
- the chart indexes edges by start and end string position (aka vertices);
- dot in rule RHS indicates degree of completion: $\alpha \rightarrow \beta_1 ... \beta_{i-1} \bullet \beta_i ... \beta_n$
- *active* edges (aka *incomplete* items) — partial RHS: $[1, 2, \text{VP} \rightarrow \text{V} \bullet \text{NP}]$;
- *passive* edges (aka *complete* items) — full RHS: $[1, 3, \text{VP} \rightarrow \text{V NP} \bullet]$;

The Fundamental Rule

$$[i,\ j,\ \alpha \rightarrow \beta_1 ... \beta_{i-1} \bullet \beta_i ... \beta_n] + [j,\ k,\ \beta_i \rightarrow \gamma^+ \bullet]$$
$$\mapsto [i,\ k,\ \alpha \rightarrow \beta_1 ... \beta_i \bullet \beta_{i+1} ... \beta_n]$$

An Example of a (Near-)Complete Chart

	1	2	3	4	5
0	NP → NP • PP S → NP • VP NP → kim •				S → NP VP •
1		VP → V • NP V → adored •	VP → VP • PP VP → V NP •		VP → VP • PP VP → VP PP • VP → V PP •
2			NP → NP • PP NP → snow •		NP → NP • PP NP → NP PP •
3				PP → P • NP P → in •	PP → P NP •
4					NP → NP • PP NP → oslo •

$_0$ *Kim* $_1$ *adored* $_2$ *snow* $_3$ *in* $_4$ *Oslo* $_5$

(Even) More Active Edges

	0	1	2	3
0	S → • NP VP NP → • NP PP NP → • kim	S → NP • VP NP → NP • PP NP → kim •		S → NP VP •
1		VP → • VP PP VP → • V NP V → • adored	VP → V • NP V → adored •	VP → VP • PP VP → V NP •
2			NP → • NP PP NP → • snow	NP → NP • PP NP → snow •
3				

- Include all grammar rules as *epsilon* edges in each *chart*$_{[i,i]}$ cell.
- after initialization, apply *fundamental rule* until fixpoint is reached.

Our ToDo List: Keeping Track of Remaining Work

The Abstract Goal

- Any chart parsing algorithm needs to check all pairs of adjacent edges.

A Naïve Strategy

- Keep iterating through the complete chart, combining all possible pairs, until no additional edges can be derived (i.e. the fixpoint is reached);
- frequent attempts to combine pairs multiple times: deriving ‘duplicates’.

An Agenda-Driven Strategy

- Combine each pair exactly once, viz. when both elements are available;
- maintain *agenda* of new edges, yet to be checked against chart edges;
- new edges go into agenda first, add to chart upon retrieval from agenda.

Backpointers: Recording the Derivation History

	0	1	2	3
0	2: S → • NP VP 1: NP → • NP PP 0: NP → • kim	10: S → 8 • VP 9: NP → 8 • PP 8: NP → kim •		17: S → 8 15 •
1		5: VP → • VP PP 4: VP → • V NP 3: V → • adored	12: VP → 11 • NP 11: V → adored •	16: VP → 15 • PP 15: VP → 11 13 •
2			7: NP → • NP PP 6: NP → • snow	14: NP → 13 • PP 13: NP → snow •
3				

- Use edges to record derivation trees: backpointers to daughters;
- a single edge can represent multiple derivations: backpointer sets.

Ambiguity Packing in the Chart

General Idea

- Maintain only one edge for each α from i to j (the ‘representative’);
- record alternate sequences of daughters for α in the representative.

Implementation

- Group passive edges into *equivalence classes* by identity of α, i, and j;
- search chart for existing equivalent edge (h, say) for each new edge e;
- when h (the ‘host’ edge) exists, *pack* e into h to record equivalence;
- e *not* added to the chart, no derivations with or further processing of e;

$\rightarrow$ *unpacking* multiply out all alternative daughters for all result edges.

Chart Elements: The Edge Structure

`#[`*id* `:` `(`*i*–*j*`)` α `-->` *edge*$_1$ ... *edge*$_i$ `.` β_{i+1} ... β_n $\{$ *alternate*$_1$ ... *alternate*$_n$ $\}^*$ `]`

Components of the *edge* Structure

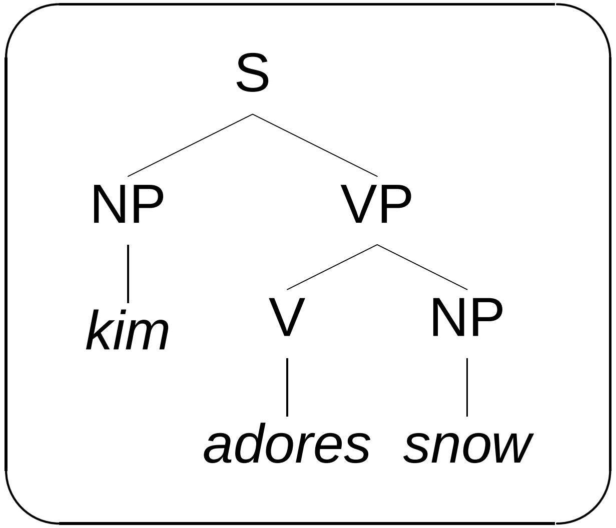
- *id* unique edge identifier (automatically assigned my `make-edge()`);
- *i* and *j* starting and ending string index (chart vertices) for this edge;
- α category of this edge (from the set C of non-terminal symbols);
- *edge*$_1$ ... *edge*$_i$ (list of) daughter edges (for β_1 ... β_i) instantiated so far;
- β_{i+1} ... β_n (list of) remaining categories in rule RHS to be instantiated;
- *alternate*$_1$ ... *alternate*$_n$ alternative derivation(s) for α from i to j;

$\rightarrow$ implemented using `defstruct()` (plus suitable pretty printing routine).

Background: Trees as Bracketed Sequences

- Trees can be encoded as sequences (*dominance* plus *precedence*):

```
(S (NP kim)
   (VP (V adored)
       (NP snow)))
```



- the `first()` element (at each level) represents the tree root (or mother);
- all other elements (i.e. the `rest()`) correspond to immediate daughters.

Ambiguity Resolution Remains a (Major) Challenge

The Problem

- With broad-coverage grammars, even moderately complex sentences typically have multiple analyses (tens or hundreds, rarely thousands);
- unlike in grammar writing, exhaustive parsing is useless for applications;
- identifying the 'right' (intended) analysis is an 'AI-complete' problem;
- inclusion of (non-grammatical) sortal constraints is generally undesirable.

Typical Approaches

- Design and use statistical models to select among competing analyses;
- for string S, some analyses T_i are more or less likely: maximize $P(T_i|S)$;

$\rightarrow$ Probabilistic Context Free Grammar (PCFG) is a CFG plus probabilities.

Probability Theory and Linguistics?

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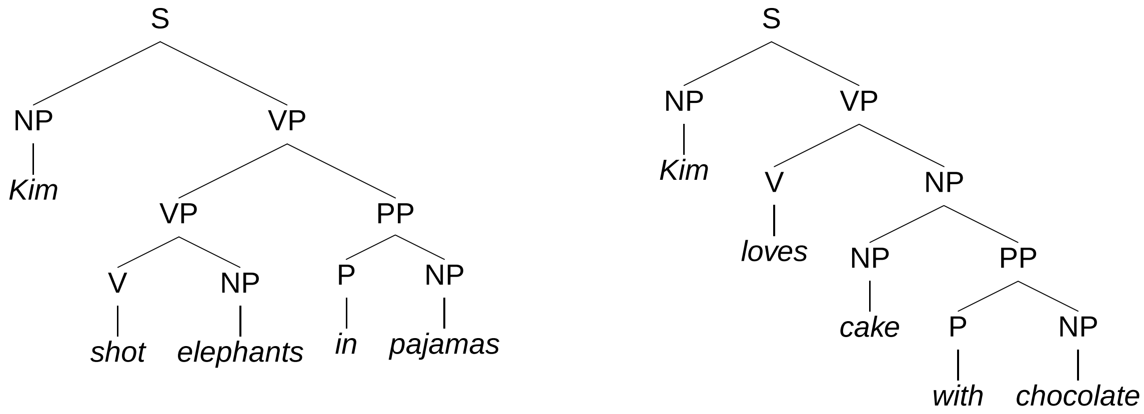
Every time I fire a linguist, system performance improves. (Fredrick Jelinek, 1980s)

A (Simplified) PCFG Estimation Example



P(RHS|LHS)	**CFG Rule**
	S → NP VP
	VP → VP PP
	VP → V NP
	PP → P NP
	NP → NP PP
	VP → V

- Estimate rule probability from observed distribution;

→ conditional probabilities:

$$P(\mathrm{RHS}|\mathrm{LHS}) = \frac{C(\mathrm{LHS}, \mathrm{RHS})}{C(\mathrm{LHS})}$$